The two-dimensional Dirac equation for a function ψDFO2():DFO2()Û^2 ⊘ Ù^2 is given by

$$\partial_t\psi(t,x) \;=\; [-\alpha(\partial_x - iA_1(t,x)) - i.m\beta + i.A_0(t,x)]\,\psi(t,x),\ t \text{ Œ } \hat{U},\ x \text{ Œ } \hat{U},\ \text{(XX)}$$

in a system of physical units in which the light velocity c and Planck's constant h are equal to one. The 2∞2 matrices α and β are hermitian with $\alpha^2 = \beta^2 = 1$ and $\alpha\beta + \beta\alpha = 0$. Both A_0 and A_1 are real valued functions on Û^2. The number m represents the rest mass of the particle whose state is associated with the function ψ.
If for the moment we suppose that the functions A_0 and A_1 are both identically zero on Û^2, then equation (XX) becomes

$$\partial_t\psi(t,x) \;=\; [-\alpha\partial_x - i.m\beta]\,\psi(t,x),\text{DFO24()}t \text{ Œ } \hat{U},\ x \text{ Œ } \hat{U}. \qquad \text{(XY)}$$

On any space L^p(Û^2,Ù^2), $1 \le p <$ •, the operator $-\alpha\partial_x$ is associated with a continuous, uniformly bounded group $S_p(t)$, t Œ Û, of linear transformations on L^p(Û^2,Ù^2): there exists a 2∞2 unitary matrix U such that $U\alpha U$SUP6(–1) = B(ACO2HS3(1,0,0,–1)), so that $US_p(t)U$SUP6(–1)$\phi(x) = (\phi_1(x+t),\phi_2(x-t))$ for ϕ Œ L^p(Û^2,Ù^2). For p = •, we have continuity for the weak*-topology $\sigma(L^•$(Û^2,Ù^2),L^1(Û^2,Ù^2)). Each of the component functions ϕ_1 and ϕ_2 is transformed according to the action of a translation in Û^2.
It follows from the Trotter product formula [K] that the operator $-\alpha\partial_x - i.m\beta$ is also associated with with a uniformly bounded group of operators on $L^•$(Û^2,Ù^2), so Theorem XYZ shows that there are countably additive operator valued measures associated with equation (XY) and solutions to equation (XX) can be represented by integrals with respect to these measures.
Similarly, the wave equation in two space-time dimensions is

$$\partial_t^{\,2}\psi(t,x) \;= c^2\,\partial_x^{\,2}\psi(t,x) \quad,\quad \psi(0,x) = f(x) \quad,\quad \psi_t(0,x) = g(x)\,,\ x \text{ Œ } \hat{U},\ t \ge 0.$$

Let $v(t,x) = \partial_t u(t,x) - c\,\partial_x u(t,x)$, $\partial_t v(t,x) + c\,\partial_x v(t,x) = 0$, with

$v(0,x) \;=\; g(x) - cf\,'(x),\ u(x) = f(x)$, for all x Œ Û.

On setting ϕ = B(A(u,v)), the equation becomes the first order system

$\partial_t\phi = A\,\partial_x\phi + iQ\phi$, Q = B(A(0 -i,0 0)), A = B(A(c 0,0 -c)),

with the initial condition $\phi(0,x)$ = B(A($f(x)$,$g(x) - cf\,'(x)$)), x Œ Û.
The operator $A\partial_x\phi + iQ$ is also associated with with a uniformly bounded group of operators on $L^•$(Û^2,Ù^2), so perturbations to the wave eqaution may also be represented in terms of path integrals.

Similar considerations apply to the $N\infty N$ hyperbolic system of the first order

$$\partial_t \psi(t,x) \quad = \quad \Big[\mathrm{ISU}\big(l{=}1{,}d,\ \big)P_l\big(\partial_{x\mathrm{SDO2}(i)} - iA_l(t,x)\big) + iQ + iV(t,x)\ \Big]\psi(t,x),\ 0 < t < T,\ x \text{ Œ } \mathbf{R}^d, \qquad \text{(XXA)}$$

where $0 < T < \bullet$, and P_l, $1 \le l \le d$, and Q are constant $N\infty N$-matrices, and $A_l(t,x)$, $1 \le l \le d$, and $V(t,x)$, $0 \le t \le T$, x Œ $\mathbf{R}^d$ are real-valued functions. The function ψ has values in $\mathbf{C}^N$. It is assumed that P_l , $1 \le l \le d$ have only real eigenvalues, and that they are simultaneously diagonalizable. The path space measures associated with the first order hyperbolic system (XXA) were first considered by T. Ichinose [Ich] who examined properties of the fundamental solution of the system (XXA).

The alternate viewpoint using the Trotter product formula outlined above was formulated in [Jeff] as an application of Theorem ZZZ. The essence of this approach is that there is a collection of dynamical systems, represented by translations along the x_i-axes, $1 \le i \le d$ for the equations (XXA), that act independently on components of the state vector prior to suffering a *mixing* of components via a semigroup of operators, for example, the semigroup generated by the constant matrix iQ in equation (XXA).

The operator $\mathrm{ISUIN}\big(l{=}1{,}d,\ \big)P_l\partial_{x\mathrm{SDO2}(i)}$ in (XXA) can be written more suggestively as $\mathrm{IIN}(_{K},\ )\mathrm{ISUIN}\big(l{=}1{,}d,\ \big)\lambda_i\partial_{x\mathrm{SDO2}(i)}\ \mathrm{d}R(\lambda_1,\ldots,\lambda_d)$. The matrices P_l , $1 \le l \le d$ are simultaneously diagonalizable, so there exists a discrete spectral measure R acting on Ù^N such that $P_i =$ $\mathrm{IIN}(_{K},\ )\lambda_i\ \mathrm{d}R(\lambda_1,\ldots,\lambda_d)$, for each $1 \le l \le d$; the set K is the joint spectrum of the system of matrices $(P_1,\ldots,P_d)$.

Our aim now is to apply these ideas in the general setting of a direct sum of dynamical systems over a single measure space $(\Sigma,$E$,\mu)$: